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$$\begin{aligned}
&= a \int_0^a \int_v^{a-v} \log \left[\frac{\sqrt{[(a-u)^2 + (a-v)^2]} - (a-u)}{\sqrt{[u^2 + (a-u)^2]} + u} \right] dv \, du \\
&\quad + a \int_a^a \int_{a-v}^u \log \left[\left(\frac{v}{a-v} \right) \left(\frac{\sqrt{[(a-u)^2 + (a-v)^2]} - (a-u)}{\sqrt{[(a-u)^2 + v^2]} + a-u} \right) \right] du \, dv \\
&\quad - \int_0^a \int_0^u \left[(a-u) \log \left(\frac{u}{a-u} \right) \left(\frac{\sqrt{[(a-u)^2 + (a-v)^2]} + (a-v)}{\sqrt{[u^2 + v^2]} + v} \right) \right] \\
&\quad + v \log \left[\left(\frac{v}{a-v} \right) \left(\frac{\sqrt{[(a-u)^2 + (a-v)^2]} - (a-u)}{\sqrt{[u^2 + v^2]} - u} \right) \right] du \, dv \\
&= 2a \int_0^a \left[v \log \left(\frac{\sqrt{[(a-v)^2 + v^2]} + v}{a-v} \right) - (a-v) \log(\sqrt{2}+1) + (a-v) \sqrt{2} \right. \\
&\quad \left. - \sqrt{[(a-v)^2 + v^2]} \right] dv \\
&\quad + 2a \int_a^a \left[2(a-u) \log(\sqrt{2}+1) - (a-u) \log \{ \sqrt{[(a-u)^2 + u^2]} + u \} \right. \\
&\quad \left. - u \log \left(\frac{\sqrt{[(a-u)^2 + u^2]} + (a-u)}{u} \right) + (a-u) \log(a-u) \right] du \\
&\quad + \frac{1}{2} \int_0^a \left[2au - u^2 - a^2 \sqrt{2} + (a-u) \sqrt{[(a-u)^2 + a^2]} \right. \\
&\quad \left. - a^2 \log \left(\frac{\sqrt{[(a-u)^2 + a^2]} - (a-u)}{a(\sqrt{2}-1)} \right) \right] du \\
&= \frac{a^3}{12} [9(\sqrt{2}+1) \log(\sqrt{2}+1) - 4 - 5\sqrt{2}].
\end{aligned}$$

$$\therefore \Delta = \frac{N}{D} = \frac{a}{3} \left(\frac{9(\sqrt{2}+1) \log(\sqrt{2}+1) - 4 - 5\sqrt{2}}{\pi + 2 \log 2} \right)$$

NOTE.—This solution differs from Mr. Heaton's in that Dr. Zerr assumes the lines to be secant lines, that is, lines whose extremities lie on the sides of the square. Mr. Heaton intended the lines to be radial lines, that is, lines the extremities of which are the random point and points on the sides of the square. In both solutions, the same law of distribution has been assumed. ED. F.

188. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Find the average length of a hole at random through a given (a) sphere, (b) cube.

Solution by G. B. M., ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

(a) Let one point remain fixed and be the origin. Then $x^2 + y^2 + z^2 = 2ax = d^2$ is the equation to the sphere of radius a .

Let $\sqrt{[2ax-x^2-y^2]}=z'$, $\sqrt{[2ax-x^2]}=y'$, Δ =average length.

$$\begin{aligned} \therefore \Delta &= \frac{\int_0^{2a} \int_0^{y'} \int_0^{z'} d \, dx \, dy \, dz}{\int_0^{2a} \int_0^{y'} dx \, dy \, dz} = \frac{\int_0^{2a} \int_0^{y'} \sqrt{(2a-x-x^2-y^2)} \, dx \, dy}{\int_0^{2a} \int_0^{y'} \sqrt{[2ax-x^2-y^2]} \, dx \, dy} \\ &= \frac{\sqrt{[2a]} \int_0^{2a} [2ax-x^2] \sqrt{x} \, dx}{\int_0^{2a} \sqrt{[2ax-x^2]} \, dx} = \frac{\frac{64a^4}{35}}{\frac{4a^3}{3}} = \frac{48a}{35}. \end{aligned}$$

(b) Let the coordinates of ingress and egress of the hole be for opposite faces $(x, y, 0)$; (u, v, a) , and for adjacent faces $(x, y, 0)$; and (a, v, z) .

$\sqrt{[(x-u)^2+(y-v)^2+a^2]}=l$, $\sqrt{[(a-x)^2+(y-v)^2+z^2]}=l'$, average length= p .

$$\begin{aligned} \therefore p &= \frac{\int_0^a \int_0^a \int_0^x \int_0^y l \, dx \, dy \, du \, dv + \int_0^a \int_0^a \int_0^a \int_0^y l' \, dx \, dy \, dz \, dv}{\int_0^a \int_0^a \int_0^x \int_0^y dx \, dy \, du \, dv + \int_0^a \int_0^a \int_0^a \int_0^y dx \, dy \, dz \, dv} \\ &= \frac{4}{3a^4} \left[\int_0^a \int_0^a \int_0^x \int_0^y l \, dx \, dy \, du \, dv + \int_0^a \int_0^a \int_0^a \int_0^y l' \, dx \, dy \, dz \, dv \right] \\ &= \frac{2}{3a^4} \int_0^a \int_0^a \int_0^x \left[y \sqrt{[(x-u)^2+y^2+a^2]} \right. \\ &\quad \left. + [a^2+(x-u)^2] \log \frac{y+\sqrt{[(x-u)^2+y^2+a^2]}}{\sqrt{[a^2+(x-u)^2]}} \right] dx \, dy \, du \\ &\quad + \frac{2}{3a^4} \int_0^a \int_0^a \int_0^a \left[y \sqrt{[(a-x)^2+y^2+z^2]} \right. \\ &\quad \left. + [z^2+(a-x)^2] \log \frac{y+\sqrt{[(a-x)^2+y^2+z^2]}}{\sqrt{[z^2+(a-x)^2]}} \right] dx \, dy \, dz \\ &= \frac{2}{9a^4} \int_0^a \int_0^a \left[2xy \sqrt{[a^2+x^2+y^2]} + (3a^2y+y^3) \log \frac{x+\sqrt{[a^2+x^2+y^2]}}{\sqrt{[a^2+y^2]}} \right. \\ &\quad \left. + (3a^2x+x^3) \log \frac{y+\sqrt{[a^2+x^2+y^2]}}{\sqrt{[a^2+x^2]}} - 2a^3 \tan^{-1} \left(\frac{xy}{a \sqrt{[a^2+x^2+y^2]}} \right) \right] dx \, dy \end{aligned}$$

$$\begin{aligned}
& + \frac{2}{9a^4} \int_0^a \int_0^a 2ay \sqrt{a^2 + y^2 + z^2} + (3z^2 y + y^3) \log \frac{a + \sqrt{a^2 + y^2 + z^2}}{\sqrt{y^2 + z^2}} \\
& + (3az^2 + a^3) \log \frac{y + \sqrt{a^2 + y^2 + z^2}}{\sqrt{a^2 + z^2}} - 2z^3 \tan^{-1} \left(\frac{ay}{z\sqrt{a^2 + y^2 + z^2}} \right) \Big] dy dz \\
& = \frac{1}{36a^4} \int_0^a \left[4a^2 y^3 - 2\pi a^4 - 2\pi y^4 + 4y(2a^2 + y^2)^{\frac{3}{2}} + 2y(a^2 + y^2)^{\frac{3}{2}} \right. \\
& - 6y^3 \sqrt{a^2 + y^2} - 20a^4 \tan^{-1} \left(\frac{y}{\sqrt{2a^2 + y^2}} \right) + 4y^4 \tan^{-1} \left(\frac{y\sqrt{2a^2 + y^2}}{a^2} \right) \\
& + 4a^4 \tan^{-1} \left(\frac{\sqrt{2a^2 + y^2}}{y} \right) + 3a^4 \log 2 + 8(5a^3 y + 3ay^3) \log \frac{a + \sqrt{2a^2 + y^2}}{\sqrt{a^2 + y^2}} \\
& + 2a^2(3a^3 + 4y^2) \log \frac{y + \sqrt{a^2 + y^2}}{a\sqrt{2}} + 8(3a^4 - a^2 y^2) \log \frac{y + \sqrt{2a^2 + y^2}}{a\sqrt{2}} \Big] dy \\
& = \frac{a}{1620} [286 - 186\pi + 464\sqrt{2} - 372\sqrt{3} + 768 \log(1 + \sqrt{2}) + 1752 \log(1 + \sqrt{3}) \\
& - 936 \log 2] = \frac{2}{27} a, \text{ nearly.}
\end{aligned}$$

MISCELLANEOUS.

166. Proposed by F. H. SAFFORD, Ph. D., The University of Pennsylvania.

Several equal rectangular boxes are placed in a row with uniform intervals between the boxes and a passageway along one side of the row. Find the least width of the passageway permitting a box to be removed from the row without moving adjacent boxes. This problem arose during the construction of a room for storage batteries.

Solution by the PROPOSER.

Length of box = $PQ = a$; $\angle DAY = \phi = \cos^{-1}(b/c)$. Breadth of box = $PS = b$; $\angle RAY = \theta$. Space between boxes = $AB = c$; $OA = l = c \cot \phi$. Passage in front of boxes = $PW = h$; $OB = m = c \csc \phi$; $AR = n = c \csc \theta - b \cot \theta$.

The complete solution of this problem leads to seven cases corresponding to the possible relations between a , b , c . The desired solution is $h = PW$, and unless otherwise stated; $\phi \neq 0$. The general cases are 1°, $a > c \csc \phi$; 2°, $c \csc \phi > a > c$; 3°, $c > a > c \sin \phi$; 4°, $c \sin \phi > a > 0$.

In case 1°, Fig. 1, the box remains partly in the "space" when PW is a maximum. The axes and the auxiliary constant ϕ are determined by the position of the box when Q coincides with B , and the minimum value of y corresponds to the desired value of PW . The path of P is given by